LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **STATISTICS**

SECOND SEMESTER – **NOVEMBER 2015**

ST 2502/ST 2500 - STATISTICAL MATHEMATICS - I

Date : 28/09/2015 Dept. No. Max. : 100 Marks

Time : 09:00-12:00

SECTION A (10 x 2 = 20 Marks)

Answer ALL questions

1. Show that $\lim_{x\to 2}\frac{|x-2|}{x-2}$ does not exist.

2. What is meant by discrete distribution?

3. Define convergence of a series.

4. A discrete random variable with probability function

$$f(x) = \begin{cases} \frac{1}{x(x+1)}, & x = 1,2,\ldots\ldots \\ 0, & \text{otherwise} \end{cases}$$

Show that mean does not exist.

5. Examine the applicability of Rolle's theorem to the function
$f(x) = x^2\,(1-x)$ in the interval [0,1]

6. Find the raw moments μ_1' and μ_2' from the m.g.f of Geometric distribution

$$M_X(t) = \frac{p}{1-qe^t}$$

7. Show that the set of vectors $X_1 = (1,0,0)$, $X_2 = (0,1,0)$, $X_3 = (0,0,1)$ is linearly independent.

8. A function f is defined on R by

$$f(x) = \begin{cases} x, & \text{if } 0 \le x < 1 \\ 1, & \text{if } x \ge 1 \end{cases}$$

Find the right hand derivative at x = 1.

9. Find the inverse of the matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix}$

10. Define a symmetric matrix with an example.

SECTION B (5 X 8 = 40 Marks)

Answer any FIVE questions

11. Explain the different types of discontinuation of a function.
12 Prove that " Every function which has a finite derivative at a point is continuous at that point but not conversely.
13. Show that a monotonic increasing sequence which is bounded above is convergent.
14. Discuss the convergence of $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$.
15. State and prove Rolle's Theorem.
16. Obtain the MGF of a discrete distribution with pmf, $p(x) = pq^x$, $x = 0, 1, 2, \ldots$
Hence find the mean.
17. Examine the linear independence of the vectors (2, 4, 8), (3, 9, 27), (1, 1, 1)
18. Find the rank of a matrix A by reducing it to the Normal form

$$A= \begin{bmatrix} 1 & -1 & 3 & 6 \\ 1 & 3 & -3 & -4 \\ 5 & 3 & 3 & 11 \end{bmatrix}$$

SECTION C [2 x 20 = 40 Mark]

Answer any TWO questions

19 (a) Find the extreme points of the function $f(x) = 2x^3 - 15x^2 + 36x + 1, -\infty < x < \infty$.
(b) Find the Maclaurin's series expansion of f(x) = sin ax. (10+10)

20 (a) Discuss the convergence of $\sum_{n=1}^{\infty} \frac{1}{n^p}$ for the various values of 'p'.
(b) Verify convergence or divergence of the following series and state the test you use:
(i) $\sum_{n=1}^{\infty} \frac{(2n-1)}{n(n+1)(n+2)}$ (ii) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n}$ (10+10)

21 (a) Discuss the properties of cumulative distribution function.
(b) If the joint c.d.f of X and Y is $F(x,y) = \begin{cases} 1 - e^{-x} - e^{-y} + e^{-(x+y)}, & x > 0, y > 0 \\ 0, & otherwise \end{cases}$
Find the marginal p.d.f's of X and of Y. Are X and Y independent. (10+10)

22 (a) Find the values of ' a' so that rank ρ(A) < 3 , where A is the matrix

$$A = \begin{bmatrix} 3a-8 & 3 & 3 \\ 3 & 3a-8 & 3 \\ 3 & 3 & 3a-8 \end{bmatrix}$$

b) Find the inverse of the matrix

$$A = \begin{bmatrix} 2 & 1 & -1 \\ 0 & 2 & 1 \\ 5 & 2 & 2 \end{bmatrix}$$

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